

SUSY QCD Corrections at Two-Loop Level for $h \rightarrow bb$

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Outline

- Motivation for $h \rightarrow b\bar{b}$
- Status
- Effective theory
- Low energy theorem in QCD and SUSY QCD for the hbb coupling
- Numerical analysis
- Summary

Motivation

- Higgs is the only SM particle not yet discovered
→ analyze its mass / couplings
- E. g. Hgg and Hbb couplings are essential for various physical processes studied
- Higgs strongly couples to heavy particles
→ window for new physics through loop effects
 - $gg \rightarrow H$ possible only through quantum loop effects
 - H^+ decay does not even exist in SM

SUSY Status

SUSY effects in $h \rightarrow gg$

- 1-loop: [Djouadi, Graudenz, Spira, Zerwas '95]
- 2-loop: stop-contributions
[Aglietti, Bonciani, Degrassi, Vicini '06], [Anastasiou, Beerli, Daleo, Kunszt '06], [Mühlleitner, Spira '07]
- 2-loop: stop and gluino contributions
[Anastasiou, Beerli, Daleo '08], [Harlander, Steinhauser '04], [Degrassi, Slavich '08], [Mühlleitner, Spira '08], [Mühlleitner, Rzehak, Spira '08]

SUSY effects in $h \rightarrow b\bar{b}$

- 1-loop: [Dabelstein '95], [Coarasa, Jimenez, Sola '96], [Eberl, Hidaka, Kraml, Majerotto, Yamada '00]
- 1-loop, large $\tan \beta$: [Guasch, Häfliger, Spira '03] [Carena, Garcia, Nierste, Wagner '00]
- 2-loop: [Noth, Spira '08]

Effective theory approach

Weinberg 1980; Ovrut, Schnitzer 1981

(SUSY) QCD effects:

- Use effective Lagrangian containing
 - Higgs bosons
 - Light fermions
 - Gluon
 - Loop effects of heavy particles are encoded in coefficients of EFT operators
 - QCD: top quark
 - SQCD: top quark, squarks, gluino
- ⇒ Heavy particles are “integrated out”

Full Theory: (SUSY) QCD

$$\xrightarrow{m_i \rightarrow \infty}$$

Effective Theory: 5-flavor QCD

Effective Lagrangian (SM)

Inami, Kubota, Okada 1983; Djouadi, Spira, Zerwas 1991; Chetyrkin, Kniehl, Steinhauser 1997

$$\mathcal{L}_{\text{eff}}^{\text{Yuk}} = -\frac{H^0}{v} \sum_{i=1}^5 C_i^0 \mathcal{O}'_i$$

where the bare operators are

$$\mathcal{O}'_1 = (G_{\mu\nu}^{0',a})^2,$$

$$\mathcal{O}'_2 = \sum_{i=1}^{n_l} m_{q_i}^{0'} \bar{\psi}_{q_i}^{0'} \psi_{q_i}^{0'},$$

$$\mathcal{O}'_3 = \sum_{i=1}^{n_l} \bar{\psi}_{q_i}^{0'} \left[\frac{i}{2} (\vec{D}^{0'} - \overleftarrow{D}^{0'}) - m_{q_i}^{0'} \right] \psi_{q_i}^{0'},$$

plus other unphysical Operators $\mathcal{O}'_4, \mathcal{O}'_5$ that involve ghost interactions.

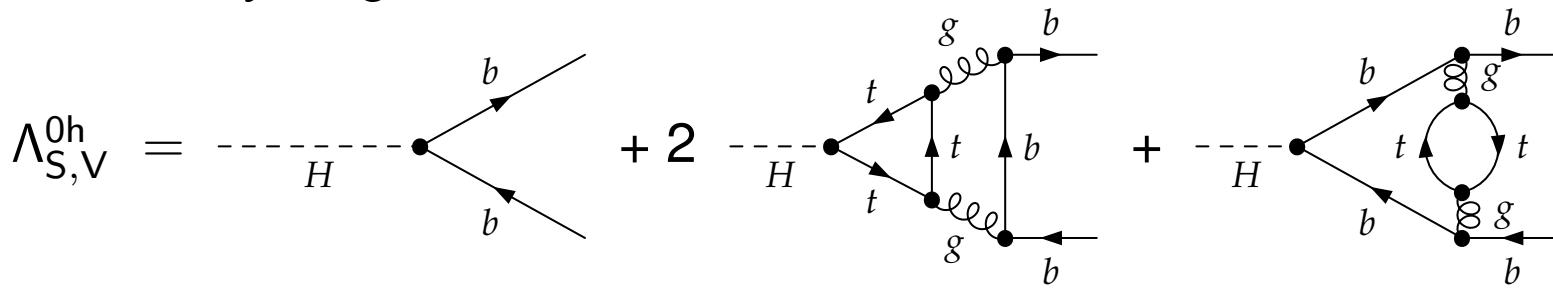
→ Coefficients $C_i^0 = C_i^0(\alpha_s^0, m_t^0, \mu)$ contain effects of heavy particle

Matching the effective theory to the full theory

Consider $Hb\bar{b}$ correlator in both theories.

(i) EFT: tree level diagrams generated by $\mathcal{O}_2, \mathcal{O}_3$

(ii) Full theory diagrams:

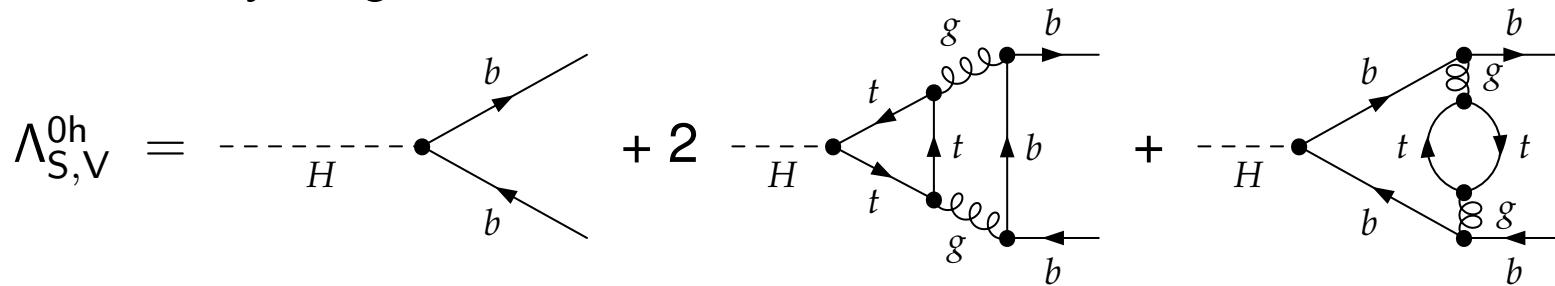


Matching the effective theory to the full theory

Consider $Hb\bar{b}$ correlator in both theories.

(i) EFT: tree level diagrams generated by $\mathcal{O}_2, \mathcal{O}_3$

(ii) Full theory diagrams:



Do matching for scalar and vector part separately:

$$\zeta_m^0 \zeta_2^0 (C_2^0 - C_3^0) = \Lambda_S^{0h} (p_i^2 = 0)$$

$$\zeta_2^0 C_3^0 = \Lambda_V^{0h} (p_i^2 = 0)$$

with the quark wave function and mass decoupling relations

$$\psi_q^{0\prime} = \sqrt{\zeta_2^0} \psi_q^0, \quad m_q^{0\prime} = \zeta_m^0 m_q^0$$

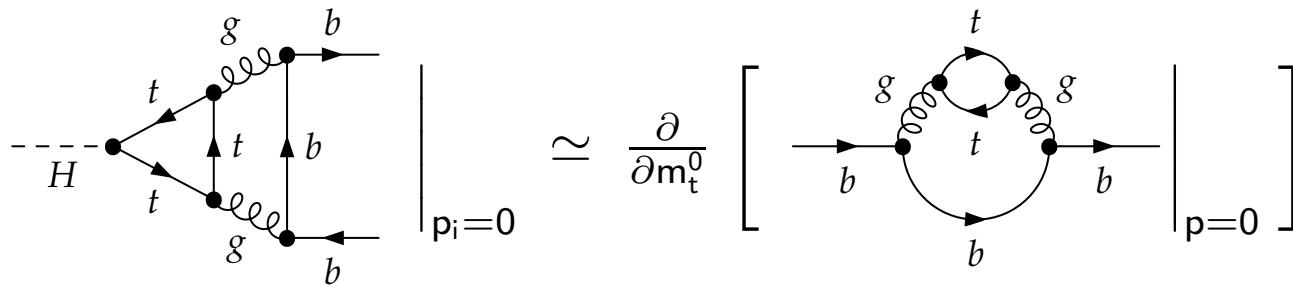
Low energy theorem for Higgs interactions

Shifman, Vainshtein, Voloshin, Zakharov 1979; Kniehl, Spira 1995; Kilian 1995;

Chetyrkin, Kniehl, Steinhauser 1997

Easiest example:

Exploit that Higgs coupling is proportional to heavy particle mass:



⇒ Vertex amplitudes $\Lambda_S^{0h}(0)$ and $\Lambda_V^{0h}(0)$ can be obtained from light quark propagator via differentiation.

⇒ Limit $M_H \ll m_t$

Low energy theorem for Higgs interactions

Shifman, Vainshtein, Voloshin, Zakharov 1979; Kniehl, Spira 1995; Kilian 1995;
Chetyrkin, Kniehl, Steinhauser 1997

In general:

Amplitude of a generic particle configuration X plus Higgs can be obtained from derivatives of X w.r.t. the Higgs field H :

$$\mathcal{A}(X, H) \Big|_{p_H=0} \simeq \frac{d}{dH} \mathcal{A}(X_H) \Big|_{H=0}$$

where X_H is an amplitude “depending on H ” via its parameters.

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where X_H is an amplitude “depending on H ” via its parameters.

E.g. SM Yukawa Lagrangian: $\mathcal{L}_{\text{Yuk}} = -\lambda_t \bar{t}^L t^R \phi^0{}^*$ with $\phi^0 = \frac{1}{\sqrt{2}}(v + H + i\chi)$
 $\Rightarrow m_t^0(H) = \lambda_t / \sqrt{2}(v + H)$

$$\frac{d}{dH} \mathcal{A}(X_{m_t^0(H), H}) \Big|_{H=0} = \left[\frac{\lambda_t}{\sqrt{2}} \frac{\partial}{\partial m_t^0} + \frac{\partial}{\partial H} \right] \mathcal{A}(X_{m_t^0, H}) \Big|_{H=0}$$

SUSY QCD low energy theorem

Degassi, Slavich, Zwirner 2001

SUSY QCD: (Bare) parameters that depend on neutral Higgs fields H_1^0, H_2^0 :

- Heavy quark mass: $m_t^2 = \lambda_t^2 |H_2^0|^2$

- Squark masses:

$$m_{\tilde{t}_{1,2}}^2 = \frac{1}{2} \left[(M_L^2 + M_R^2) \pm \sqrt{(M_L^2 - M_R^2)^2 + 4|\tilde{X}|^2} \right], \quad m_{\tilde{b}_{1,2}}^2 = \dots$$

$$\text{with } M_L^2 = M_Q^2 + \lambda_t^2 |H_2^0|, \quad M_R^2 = M_U^2 + \lambda_t^2 |H_2^0|, \quad \tilde{X} = \lambda_t (A_t H_2^0 - \mu_{\text{SUSY}} H_1^{0*})$$

- Squark mixing angles:

$$\sin 2\theta_t = \frac{2|\tilde{X}|}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}, \quad \sin 2\theta_b = \dots$$

- $M_Q, M_{U,D}$: soft SUSY breaking mass parameters
- A_q : trilinear coupling
- μ_{SUSY} : Higgs-Higgsino bilinear coupling

SUSY QCD low energy theorem

Mass eigenstates h, H from neutral Higgs fields H_1^0, H_2^0 :

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix}$$

SUSY QCD low energy theorem

Bare 5-flavor QCD effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \frac{m_b^{0'}}{v} \frac{s_\alpha}{c_\beta} (C_{2,1}^0 - \frac{1}{\tan\alpha\tan\beta} C_{2,2}^0) h \bar{\psi}_b^{0'} \psi_b^{0'} + \dots$$

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SUSY QCD coefficients:

$$C_{2,1}^0 = 1 + \hat{D}_1 \ln \zeta_m^0 = 1 - \frac{\hat{D}_1 \Sigma_S^{0h}}{1 - \Sigma_S^{0h}} - \frac{\hat{D}_1 \Sigma_V^{0h}}{1 + \Sigma_V^{0h}}$$

$$C_{2,2}^0 = \hat{D}_2 \ln \zeta_m^0 = - \frac{\hat{D}_2 \Sigma_S^{0h}}{1 - \Sigma_S^{0h}} - \frac{\hat{D}_2 \Sigma_V^{0h}}{1 + \Sigma_V^{0h}}$$

SUSY QCD low energy theorem

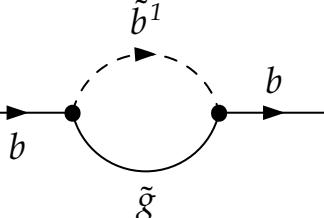
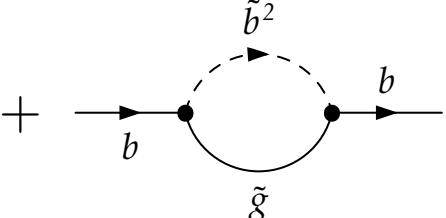
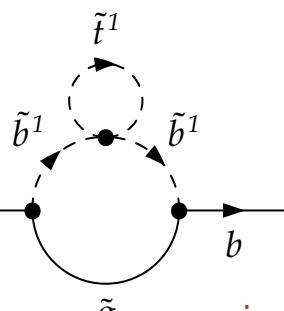
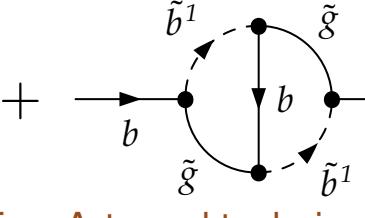
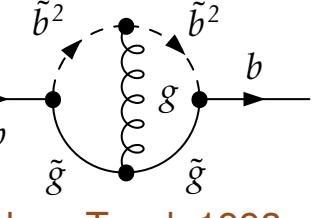
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$$C_{2,2}^0 = \hat{D}_2 \ln \zeta_m^0 = - \frac{\hat{D}_2 \Sigma_S^{0h}}{1 - \Sigma_S^{0h}} - \frac{\hat{D}_2 \Sigma_V^{0h}}{1 + \Sigma_V^{0h}}$$

- 1-loop: $\Sigma_{S,V}^{0h,(1l)} =$  + 
- 2-loop: $\Sigma_{S,V}^{0h,(2l)} =$  +  + 
+ ...

using FeynArts and techniques by Davydychev, Tausk 1993

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Generalized derivatives:

see Degrassi, Slavich 2008 for $h \rightarrow gg$

$$\hat{D}_1 = [m_b A_b s_{2\theta_b} \hat{F}_b + 2m_b^2 \hat{G}_b] - [m_t \frac{\mu_{\text{SUSY}}}{\tan\beta} s_{2\theta_t} \hat{F}_t]$$

$$\hat{D}_2 = [m_t A_t s_{2\theta_t} \hat{F}_t + 2m_t^2 \hat{G}_t] - [m_b \mu_{\text{SUSY}} (\tan\beta) s_{2\theta_b} \hat{F}_b]$$

$$\hat{F}_q = \frac{\partial}{\partial m_{\tilde{q}_1}^2} - \frac{\partial}{\partial m_{\tilde{q}_2}^2} - \frac{4c_{2\theta_q}^2}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2} \frac{\partial}{\partial c_{2\theta_q}^2}, \quad \hat{G}_q = \frac{\partial}{\partial m_{\tilde{q}_1}^2} + \frac{\partial}{\partial m_{\tilde{q}_2}^2} + \frac{\partial}{\partial m_q^2},$$

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→ Check performed by an explicit calculation of associated 2-loop vertex diagrams ✓

Numerical analysis

“small α_{eff} ” scenario: Carena et. al. '03

$$\tan\beta = 50$$

$$M_L = M_R = 800 \text{ GeV}$$

$$A_b = A_t = -1.060 \text{ TeV}$$

$$\mu_{\text{SUSY}} = 2 \text{ TeV}$$

$$m_{\tilde{g}} = 500 \text{ GeV}$$

Scale dependence of $C_{2,1}$:

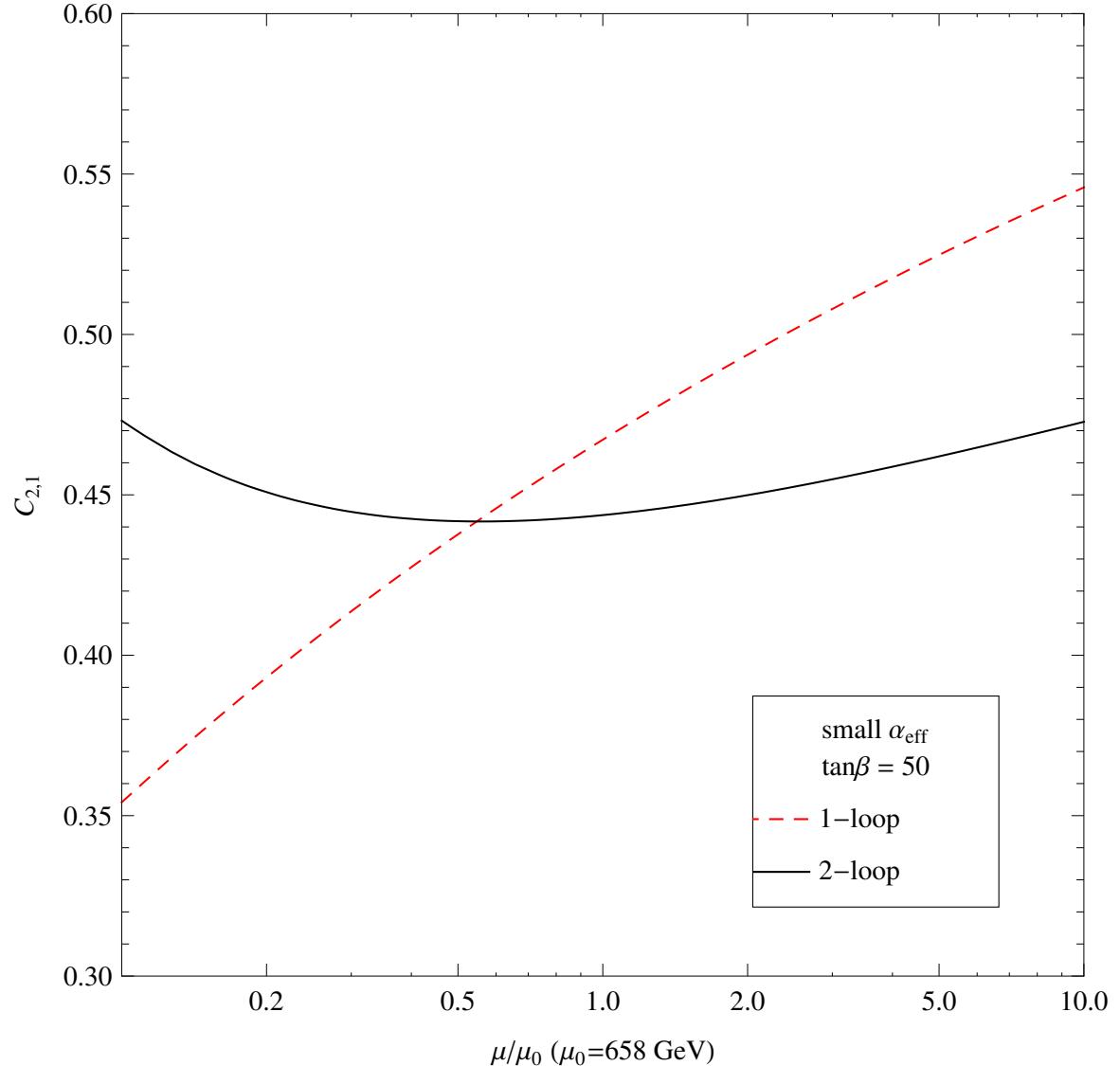
Vary μ around

$$\mu_0 = \frac{1}{3}(m_{\tilde{b}_1} + m_{\tilde{b}_2} + m_{\tilde{g}}):$$

$$0.1\mu_0 < \mu < 10\mu_0$$

- 1-loop dependence: $\sim 50\%$
- 2-loop dependence: $\sim 6\%$

\Rightarrow Significant reduction at 2-loop level



Numerical analysis

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Scale dependence of $C_{2,2}$:

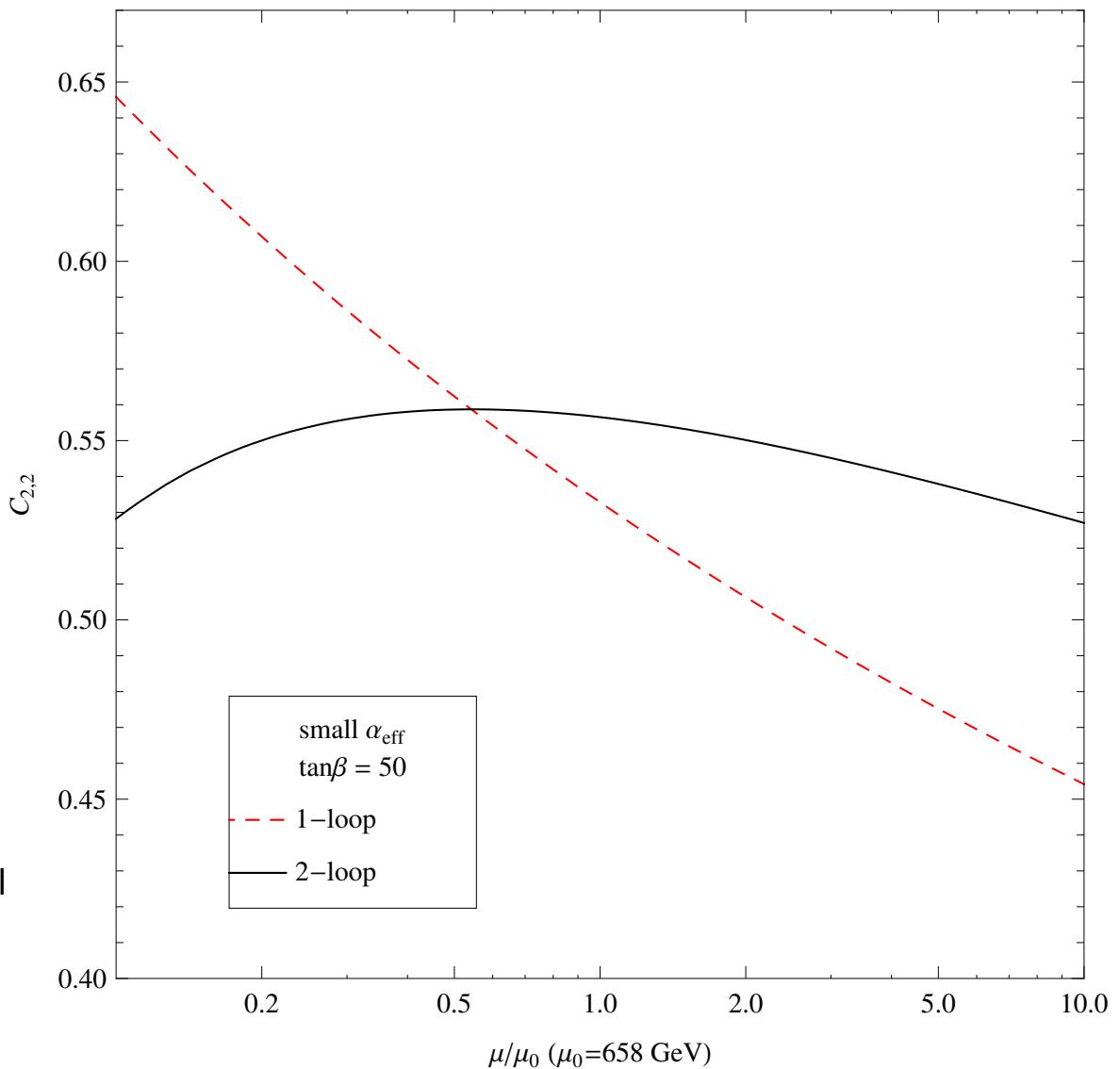
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Numerical analysis

“gluophobic” scenario: Carena et. al. ’03

$$\tan\beta = 20$$

$$M_L = M_R = 350 \text{ GeV}$$

$$A_b = A_t = -735 \text{ GeV}$$

$$\mu_{\text{SUSY}} = 300 \text{ GeV}$$

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Scale dependence of $C_{2,1}$:

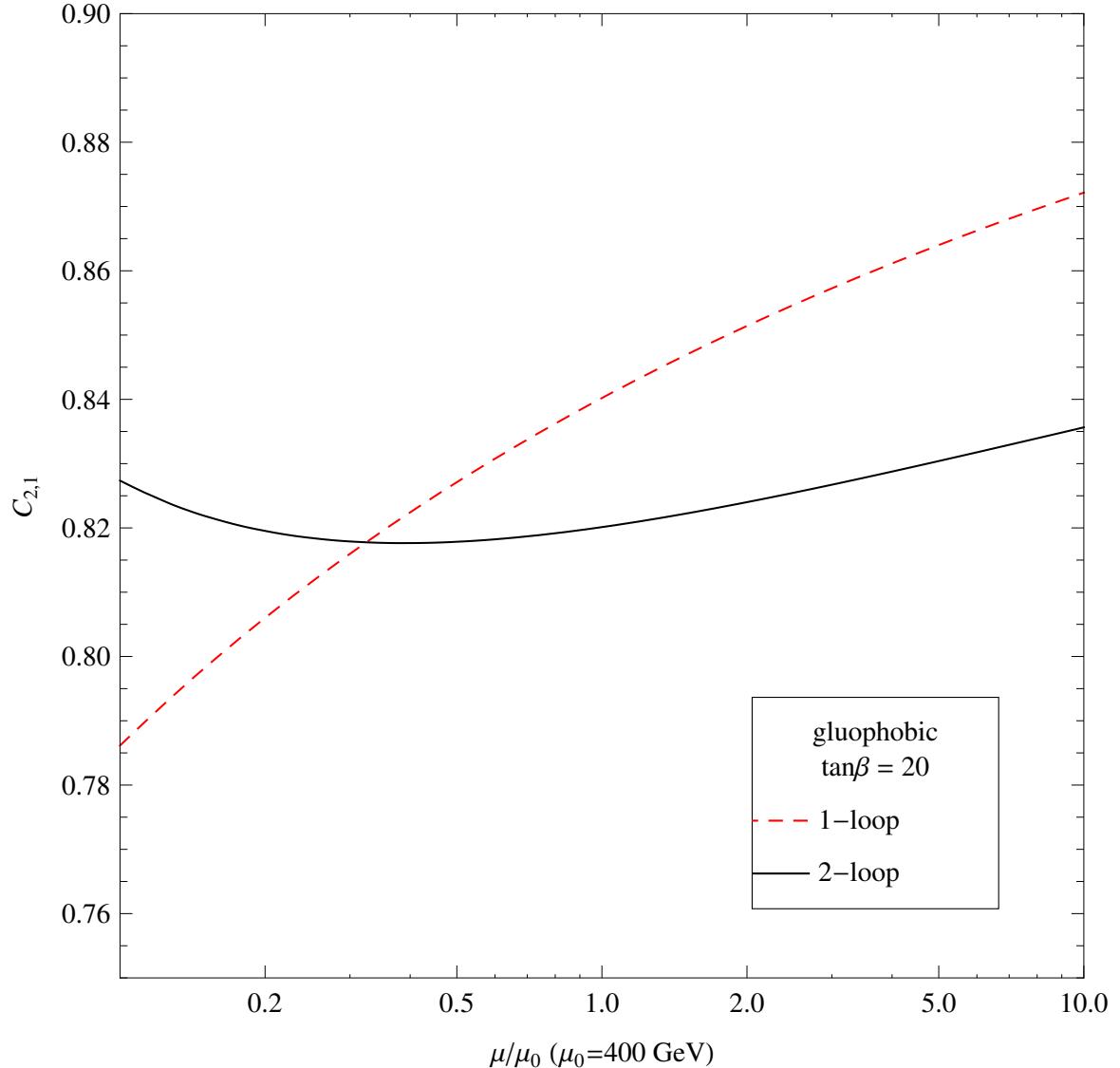
Vary μ around

$$\mu_0 = \frac{1}{3}(m_{\tilde{b}_1} + m_{\tilde{b}_2} + m_{\tilde{g}}):$$

$$0.1\mu_0 < \mu < 10\mu_0$$

- 1-loop dependence: $\sim 15\%$
- 2-loop dependence: $\sim 3\%$

⇒ Significant reduction at 2-loop level



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Scale dependence of $C_{2,2}$:

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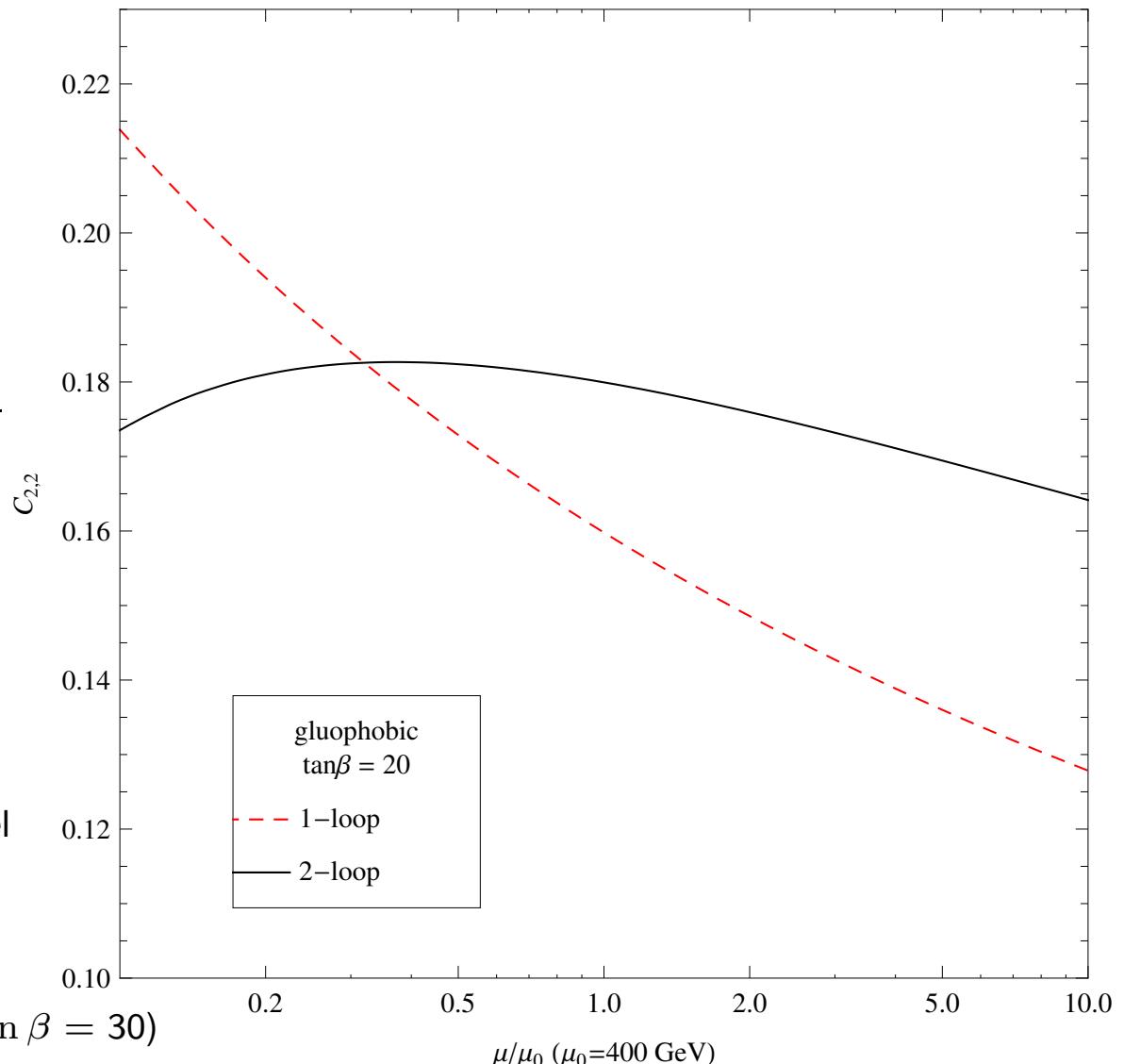
$$0.1\mu_0 < \mu < 10\mu_0$$

- 1-loop dependence: $\sim 50\%$
- 2-loop dependence: $\sim 10\%$

⇒ Significant reduction at 2-loop level

⇒ Agreement with [Noth, Spira ’08]

(“small alpha_{eff}” and “gluophobic”, $\tan\beta = 30$)



Numerical analysis

Decay width including SUSY QCD effects:

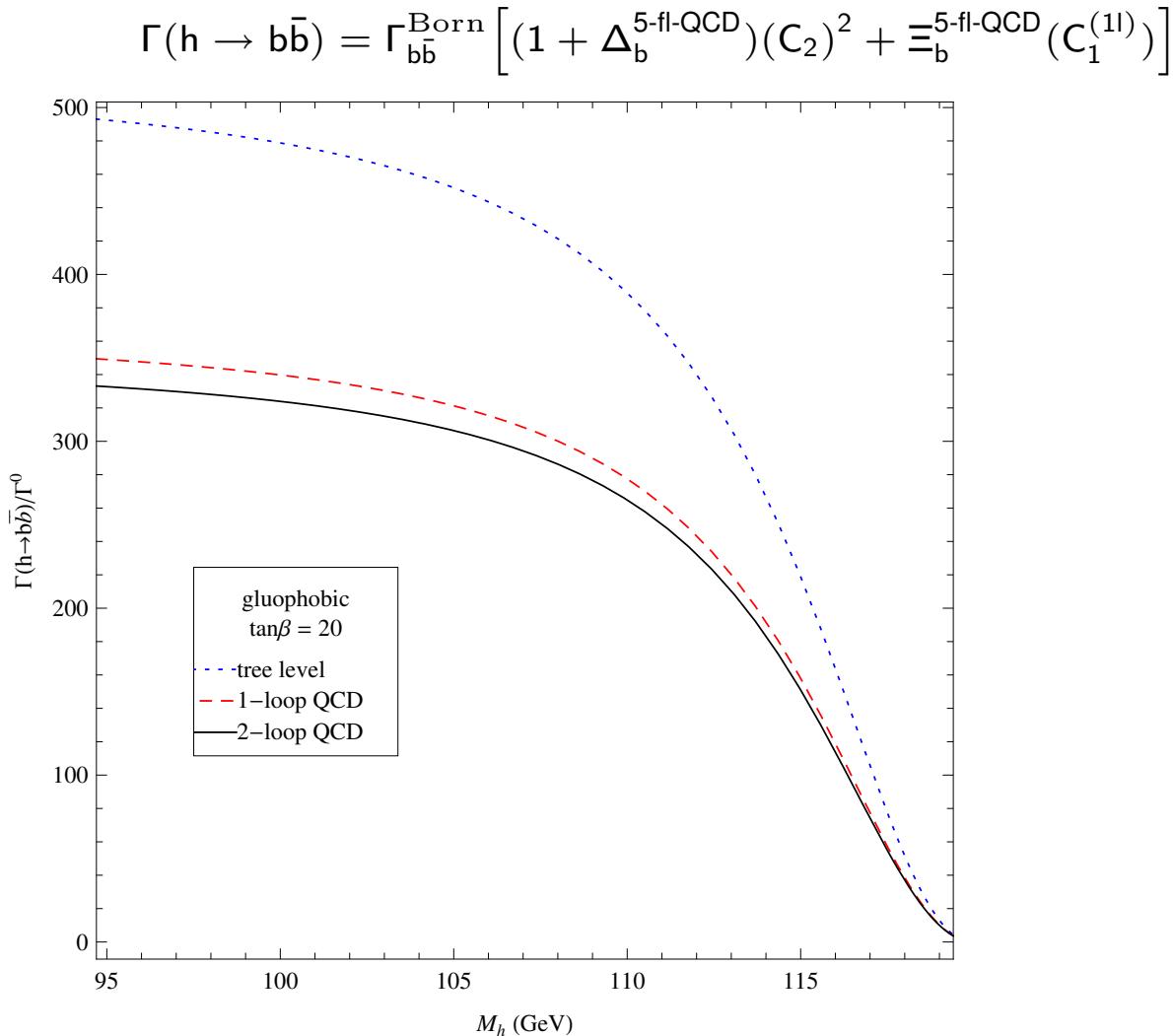
$$\Gamma(h \rightarrow b\bar{b}) = \Gamma_{b\bar{b}}^{\text{Born}} \left[(1 + \Delta_b^{\text{5-fl-QCD}})(C_2)^2 + \Xi_b^{\text{5-fl-QCD}}(C_1^{(1)}) \right]$$

where

$$\Gamma_{b\bar{b}}^{\text{Born}} = \frac{N_c G_F M_h m_b^2}{4\pi\sqrt{2}}, \quad C_2 = -\frac{s_\alpha}{c_\beta} \left(C_{2,1} - \frac{1}{\tan\alpha\tan\beta} C_{2,2} \right)$$

Numerical analysis

Decay width including SUSY QCD effects:



⇒ 2-loop corrections amount to O(8 %)

Summary

- Low energy theorem at 2-loop allows for a calculation of hbb coupling from b propagator diagrams
- Check of the 2-loop low energy theorem by a direct diagrammatic calculation
- Two independent implementations of the calculation used
- Agreement with [Noth, Spira '08]
- Significant reduction of the scale uncertainty by inclusion of 2-loop corrections
- Numerical effects reach up to $O(8\%)$, depending on the SUSY scenario chosen
- Paper in preparation